19 February 2012

Appendix W: Notes on the one-dimensional approximation to acoustic waves [1]

W.1 Introduction

This appendix is provided to document the continuing "old school" discussion of the application of one-dimensional wave theory to the Organ of Corti, a feature that is fundamentally not one dimensional either externally or in internal features.

The basic premise of this work is that Maxwell's Equations (which were developed originally in the acoustic wave context) must be satisfied in their entirety. Short-cuts designed to provide simplified textbook descriptions for pedagogy cannot be effectively adopted to explain operation of the Organ of Corti (OC) within the cochlea.

The role of dispersion appears repeatedly in the following discussions. However, the role of dispersion within the OC has not been developed in detail. It appears that dispersion plays no role in the operation of the signal channels associated with the outer hair cells (OHC). Any dispersion within the OC is easily adjusted for in the neural paths associated with the OHC. Dispersion would affect the shape of the amplitude waveforms present along Hensen's stripe and intercepted by the inner hair cells (IHC). Whether this effect is an attribute or detriment in the signal processing within the signal processing channels subsequent to the IHC has not been determined.

This appendix is closely associated with **Appendix M**, Detailed properties of the tectorial membrane, but is designed to avoid introducing extraneous material into that appendix. Section M.2.5-1 of that appendix is particularly appropriate to this discussion and is repeated below as **Section W.3.4**.

W.1.1 Potential wave environments in the Organ of Corti

There are at least four potential wave environments that could be used in the Organ of Corti. They are all described in detail by Maxwell's Laws (which were originally developed to explain acoustic mechanisms before radio waves were discovered. The equations for traveling waves within these environments always contain a term described by the product of their velocity multiplied by time.

von Bekesy wave– von Bekesy described a conceptual wave traveling within the OC and assumed to be a feature of the basilar membrane. He knew it was a slowly traveling wave, compared to a conventional acoustic wave in water, but was unaware of the technology required to achieve this speed. He therefore conceptualized the wave as the result of a continually changing substrate. The substrate exhibited a resonance frequency that changed incrementally with position (like the keys and associated strings on a piano) beginning at a high frequency near the base of the cochlea.

This conceptualization fails when one considers the variation in stiffness/mass ratio (S/M) required to achieve a frequency range of 1000:1 in human acoustic range. The S/M ratio, which is the square of the frequency ratio, must approach one million to one. Since the mass of the OC varies over a very narrow range, about 1.6:1, the stiffness would need to vary from that of diamond to gelatin over its length, and to achieve the desired sharpness of tuning, the longitudinal stiffness of the basilar membrane would necessarily be very low. Its measured stiffness over its entire length under static conditions remains close to that of gelatin

Mathematical modelers tend to arbitrarily "optimize" the S/M ratio by focusing on either the high frequency or low frequency regions of the OC separately. They have been unable to define

[1] Released: 19 February 2012

a contiguous physical model of the OC satisfying the above physical requirements. Elliot et al. demonstrate the hoops the modelers of 2011 have been jumping through to explain the operation of the BM as a resonant structure[2]. Changes in area density of the BM of 6:1 are introduced without any support or citation. This was apparently accomplished by changing the thickness of the BM by a factor of 6:1. No stated stiffness or surface tension values were provided. A low Q_{3dB} of 2.5 is apparently maintained. Elliot et al. changed to a FEM model in the same paper "Rather than develop more and more complex analytical models, . . ."`

Foucaud et al. performed a mathematical modeling at six distinct locations in the OC[3]. Interestingly, they used a Young's modulus for the BM of 2×10^8 Pa and for the TM of 1×10^3 Pa, a ratio of 200,000:1 without any justification except they used values "adjusted from literature." They employed a "no loss assumption" and with reference to their figure 3 said, "The entire real wave number tending to infinity is not realistic but is coherent with the no loss assumption."

Compression wave– Conventional compression waves could be present in both the fluids and the "solid" portions of the cochlea. These materials would support compression waves traveling on the order of 350 meters/sec. The fluids of the Organ of Corti are not known to support any traveling waves other than these compression waves.

fluid/gas interface surface wave– Traveling waves at the interface between a fluid and a gas have been widely studied, and they have been presented in introductory college texts in the physical sciences. The examples usually involve the beaches of the ocean under the assumption the waves are traveling perpendicular to the shoreline. The result is a simple one-dimensional situation where the amplitude of the wave is a function of distance from the shoreline. These surface waves subdivide into two distinct types as documented explicitly by Lighthill, 1978.

1. The major waves are conventionally described as gravity waves since gravity plays a major role in the restoring forces associated with the fluid/gas surface. These waves are subdivided into gravity waves in deep water, and gravity waves in shallow water where the depth of the water becomes a parameter.

2. A separate class of waves are the very small capillary, or wind driven, waves on the surface of a fluid/gas interface. The capillary waves travel in the direction of the wind excitation and are independent of the depth of the fluid for depths exceeding three times the wavelength of the capillary waves. The velocity and height of the capillary waves are dominated by the surface tension of the fluid. The capillary waves are a very slow traveling surface acoustic wave (SAW). In the case of the TM, they have been measured at a speed of 5-6 meters/sec.

fluid/fluid interface surface wave– Traveling waves at the interface between two fluids, forming a duct within one or both fluids, have also been extensively studied in submarine warfare and marine mammal communications over long distances. These waves are also known as internal gravity waves. However, gravity plays a very minor role if the two fluids are of nearly the same density. Since the density of both fluids is typically very close to that of water (generally the only difference is due to temperature), the amplitude and velocity of the internal surface acoustic wave is independent of gravity, and is dependent on the surface tension of the two fluids. The velocity of this surface acoustic wave can be quite slow, relative to the compression waves in the fluids.

Because of the wavelengths involved, seismic waves associated with earthquakes are generally treated as surface acoustic waves within one or more materials heterogenous solid materials acting like fluids or at a fluid/gas interface where the heterogenous material can be considered a fluid at the

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[2]Elliot, S. Mi, G. Mace, B. & Lineton, B. (2011) How many waves propagate in the cochlea? *In* Shera, C. & Olson, E. *eds*. What Fire is in Mine Ears: Progress in Auditory Biomechanics. Melville, NY: Amer. Inst Physics: pp 563-568

[3]Foucaud, S. Guilhem, M. Morlier, J. & Gourinat, Y. (2011) Harmonic response of the Organ of Corti: Resuls for wave dispersion *In* Shera, C. & Olson, E. *eds*. What Fire is in Mine Ears: Progress in Auditory Biomechanics. Melville, NY: Amer. Inst Physics: pp 569-575

wavelengths involved.

Shock waves– If a simple longitudinal wave enters a nonlinear regime, it is called a shock wave. A frequent situation generating shock waves involves fluids where the phase velocity varies considerably with frequency (wavelength) compared to the group velocity. This condition can be created by using a piston to excite a compression wave to a signal velocity exceeding the nominal compression wave velocity in that fluid. The result for a fluid/gas interface is the creation of a crest in a normal surface wave followed by chaotic conditions as the crest "breaks."

Interface waves between solids and fluids, or solids and gases will not be discussed further in this section. Shock waves will not be discussed further. They are mentioned only to make investigators aware of this overdriven condition that leads to discontinuous conditions and impossible analytical conditions. The condition is easily generated using a "speaker" closely coupled to a fluid. The instantaneous velocity of the speaker diaphragm can easily exceed the local velocity limit of the fluid.

Under linear conditions, all of the above waves can be described as traveling waves, regardless of their velocity. Each frequency component is described by an equation of the following one-dimensional form; $A = \sin\omega(t \pm. x/v)$ where A is the amplitude of the wave and x is the distance from x_0 and v is the velocity of the wave along the x-dimension. At any instant in time, the wave is a sine wave in x/λ.

A stationary wave, or standing wave, is caused by the summation of two linear traveling waves moving in opposite directions. If the two waves given above are of equal amplitude and traveling in opposite direction (one + and one –), their sum has an amplitude envelope that is given by $2A\sin 2\pi(x/\lambda)$. The two traveling waves may be of a single frequency or consist of multiple frequencies in the same proportions. If the two components are of a single frequency the resulting stationary wave will appear sinusoidal as a function of position. If the waves traveling in opposite directions have more complex frequency structures, the resulting stationary wave will be more complicated, reflecting the components present at each frequency.

Fluid/fluid interface surface waves can be in the plane of the interface boundary with an amplitude that is lateral to the direction of travel (called Lamb waves) or they can be perpendicular to the interface boundary (called Rayleigh waves).

Such traveling waves follow Maxwell's Laws, and Huygen's Principle. The result are waves radiating from a local or point source in the absence of other forces. At great distances from the source, the wave front of such waves are frequently defined as plane waves because of their great radius of curvature.

When encountering boundaries, the waves can be considered as reflected or re-radiated in accordance with the same Maxwell's Laws and Huygen's Principle. The result is usually a very complex wave structure associated with a boundary to, or a discontinuity in, the fluid environment. This is the situation found in the scale models of the fluids of the Organ of Corti reported by Tam et al (2009).

Guided surface wave–

The guided surface acoustic wave is a slow surface acoustic wave compatible with the conditions present in the progressive exponential spiral of the tectorial membrane (TM) within the Organ of Corti. This spiral is generally adjacent to the similar spiral of the basilar membrane (BM). However, it, and specifically Hensen's stripe, are not precisely aligned to the BM.

W.1.1.2 The guided topographic wave

A guided surface wave is usually guided by a variation in the stiffness or surface tension of one of the materials at the interface. The object is to change the direction of the associated plane wave without involving a reflection based on Maxwell's Laws or a regeneration involving Huygen's Principle. It is more similar to Einstein's description of bending a light beam by warping space due to the presence of a large nearby mass.

Hensen's stripe is a raised ridge along the inner edge of the liquid crystalline surface of the tectorial membrane facing the BM. As a result, it is a three-dimensional structure associated with an otherwise two-dimensional sheet, called Kimura's membrane. As noted above Kimura's membrane is formed of a liquid crystalline material. As such it is described as a biological "membrane," but structurally it is a "plate with a free edge" and no internal tension. Energy propagating along Hensen's stripe is steered along the stripe by the variation in density and local surface geometry between the liquid crystalline surface and the nearby true fluid of the Scala Media. The scalar "speed" of this steered energy, as opposed to its vectorial velocity (which is a rotating vector) is typically in the 5-6 m/sec.

As discussed elsewhere, the rotating vector associated with Hensen's stripe cannot continue along Hensen's stripe if Hensen's stripe exhibits excessive curvature. The mechanism is known as the Marcatili Effect. As a result, the energy of the wavefront above a specific frequency is shed by Hensen's stripe at a specific location along the stripe and that energy proceeds to travel as a wavefront along the flat surface of Kimura's (plate) of the tectorial membrane until the Outer Hair Cells (OHC) are encountered where a majority of the energy of the relevant frequency is absorbed..

W.1.2 Detailed characteristics of steady state waves

Physical materials can support a wide variety of vibration modes, and modern computational methods allow these vibrations to be modeled very accurately in structures of great in-homogeneity (even like the basilar membrane). However, the mathematical description of the particle motion associated with acoustic energy propagation is difficult. Many caricatures of wave motion have appeared in the literature but no one caricature has gained wide acceptance. **Figure W.1.2-1** provides a framework for discussing the modes of interest to hearing. The materials supporting the vibrations in this figure are assumed to be homogeneous and infinite in extent. This will not be true when actually exploring the operation of the cochlear partition.

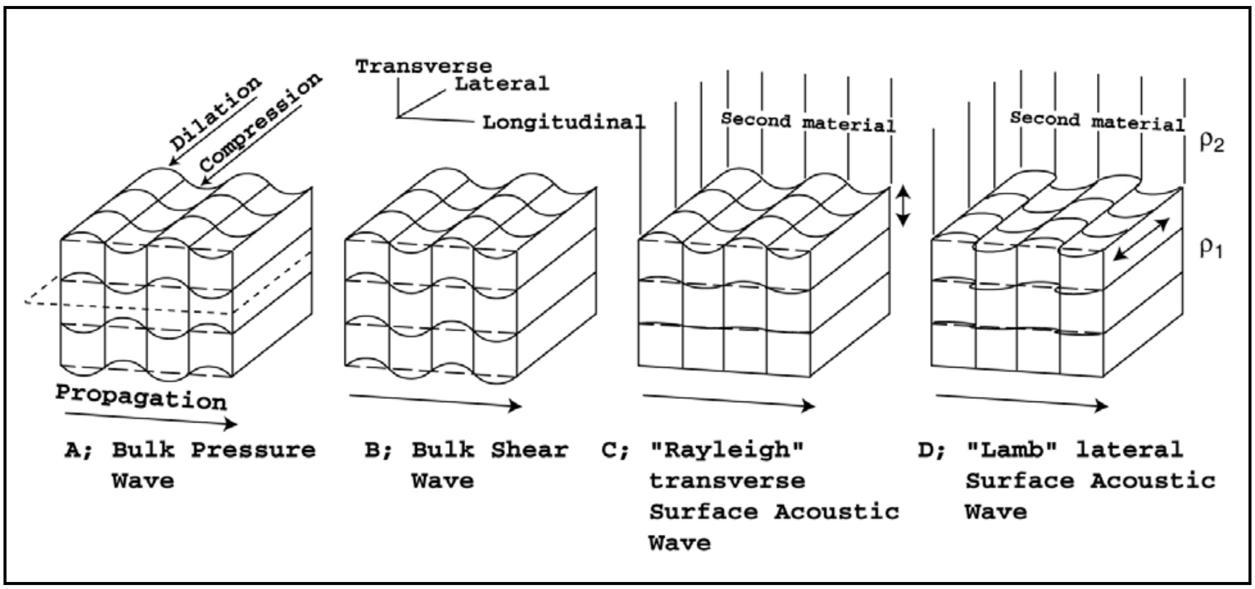

Figure W.1.2-1 Acoustic energy propagation in materials. A & B; bulk acoustic wave propagation in hree dimensional materials. A; a pressure wave. Particles each side of the line of propagation tend to move in opposite directions (out of phase). B; a shear wave. Particles each side of the line of propagation tend to move in the same direction (in phase). The pressure wave always causes a volume change with propagation whereas the shear wave does not. C; sagittal surface acoustic wave propagation at the surface between two materials. D, lateral SAW propagation at the surface between two materials. The direction of propagation is generally taken as parallel to the longitudinal axis in textbooks.

The bulk pressure wave and the bulk shear wave are shown as if they were formed within a plate of infinite extent into and out of the paper when excited by a line source along the dotted edge shown on the left. The surface acoustic wave is shown as if excited on the surface of an infinite half-sphere (sometimes called a half-space) and excited by a line source disappearing into the paper. The second material above the surface may be a vacuum or another material. The energy density associated with the surface acoustic wave attenuates with distance into the half-sphere.

A real material is three-dimensional, and the cochlear partition is a very complex three-dimensional structure. Any explanation of the propagation of acoustic energy along this structure must recognize its complexity.

When unrolled, the mammalian cochlear partition can be described in three dimensions. Generically, these dimensions are the longitudinal distance (length), lateral distance (width) and transverse distance (thickness). Energy can be propagated parallel to any of these axes, or at any angle relative to these axes.

It is difficult to represent the most common propagation mode, the pressure wave, within a material. It can be represented by a pressure field that varies periodically along the axis of propagation. This pressure field induces a related variation between compression and dilation, as shown in frame **A**, that travel in parallel with the pressure wave. This variation is not necessarily in phase with the pressure. The actual degree of compression and dilation is extremely small as suggested by the common practice of considering water and most fluids as “incompressible” from a hydraulic, but not acoustic, perspective.

The compression and dilation representation can be illustrated by two planes drawn perpendicular to the direction of propagation, and on opposite sides of the energy. These two planes tend to move toward and away from each other as if they were out of phase, frame **A**. In the absence of dissipation within the material, the amplitude of the pressure wave remains constant with distance perpendicular to the line of propagation through the material.

Alternately, the energy can propagate as a shear wave, wherein any similar pair of surfaces tend to move in the same direction as if they were in phase, frame **B**. As in the pressure wave, the amplitude of the transverse wave remains constant with distance in the absence of dissipation within the material.

In real situations, the pressure wave and transverse wave must be analyzed in the context of the boundary conditions related to the material. Because of differences in the physical parameters of the media at a boundary, it is generally impossible to sustain a pressure or shear wave in the immediate vicinity of, and parallel to, a boundary. Such an arrangement generally introduces either attenuation, reflection or refraction into the overall situation.

Lord Rayleigh first documented a special case of energy propagation at the surface between two semi-infinite media of different density, now described as a surface acoustic wave (SAW). The energy appears as a shear, or sagittal, wave and becomes concentrated at that surface (showing less intensity with distance from that surface, frame **C**. This form of propagation also exhibits a constant amplitude in the absence of dissipation in either of the materials forming the surface or the presence of other boundary conditions. The energy is stored in the deformation of the elastic super-lattice formed between these two materials.

Lamb investigated a SAW at a liquid/gas interface where the wave existed as a lateral vibration concentrated at the interface, frame **D**.

Under linear conditions, the movement of individual particles subject to acoustic stimulation are well behaved. Individual particles within a bulk material (even a fluid) exhibit negligible displacement due to the propagation of acoustic energy. The particles generally move in a small local orbit around their mean position.

Different parts of the uniquely complicated structure of the cochlear partition may support any of the above forms of vibration. However, most of the structures associated with the basilar membrane exhibit structural, elastic, and dissipative properties that will not support any of these modes.

This work has found the surface acoustic wave to play a critically important role in the operation of hearing. Because of the unique physical structure of the tectorial membrane and the density of this membrane versus the nearby endolymph fluid, the acoustic energy of hearing is found to be propagated along the surface of the membrane facing the reticular lamina in a frequency-dependent two-dimensional pattern.

Section 9.11.2 addresses the extremely serious dilemma faced by all models of the Organ of Corti assuming multiple points of resonance, uncoupled from adjacent points of resonance, along the length of the basilar membrane.

W.1.3 Characteristics of the surface of the TM

The active surface of the TM is known to be made of an acellular material, with a passive protein material underlying the surface layer. This acellular material forming the surface of the tectorial membrane, including Hensen's stripe, is a liquid crystal. Liquid crystals are unique in that they exhibit very low bulk stiffness and surface tension under static conditions but exhibit very significant stiffness and surface tension at acoustic frequencies. Such materials are known to ring like a bell for an extended period when suitably excited. The ringing suggests the loss coefficients of these materials are extremely low.

W.1.4 Modeling the guided surface acoustic wave of Hensen's stripe

After reviewing the prior art, Tam et al. finally begun to address the potential wave environments in a coiled cochlea using a sophisticated finite element model (FEM) accompanied by a laser cut acrylic physical model of a single duct (single hemisphere approach)[4]. This is a significantly more advanced approach than that reported in 2005[5]. They machined a three times larger model of a single duct Organ of Corti. Their general approach has been to follow the conventional wisdom and assume a tension membrane for the basilar membrane. The physical model and the FEM are still being improved to more readily represent the actual physical Organ of Corti. Their work has surfaced the fact that the coiled cochlea may be able to support a variety of waves, but few in an acceptable manner. Discussing these waves, they asserted, "The motions were complex and nonuniform. At high frequencies, the traveling wave was observed to cut off and not travel all the way to the apex. At intermediate frequencies, multiple cross-modes, reflected waves, traveling and standing waves were all observed." Up to the recent report, they have been focused on a tension membrane separating a viscous fluid from an air space, resulting in wave structures suggestive of a loaded fluid/gas interface. They are aware that their membrane may or may not be a good representation of the basilar membrane (that several people have demonstrated is not under tension *in-vivo*.

W.2 One-dimensional waves in fluids

W.2.1 Modeling the OC as the "classical" two-dimensional rectangular box

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[4]Tam, B. Fakhraee, A. & White, R. (2011) Coiled hydromechanical scale model of the inner ear *In* Shera, C. & Olson, E. *eds*. What fire is in mine ears: Progress in auditory biomechanics. NY: Amer. Inst. Physics pp 374-379

[5]White, R. & Grosh, K. (2005) Microengineered hydromechanical cochlear model *P Natl Acad Sci USA* vol 102, pp 1296-1301

The academic literature of hearing is dominated by references to the classical model of a cochlea generally attributed to von Bekesy. It is in essence a rigid-walled two-dimensional box assumed to extend to infinity in the third dimension, be filled with a water like fluid and be separated over most of its length in the x-dimension by the basilar membrane. The BM stops short of the end of the box to accommodate a passage known as the helicotrema. The box may or may not have an explicit window at the left end of the upper chamber to accommodate an oval window, which in fact is not found at the left end of the upper chamber.

The fact that all cochlea are coiled, and the relevance of the coiling is invariably ignored or dismissed as irrelevant to the operation of the cochlea. In fact, the curvature of the cochlea is critically important to its operation. It does not perform in the absence of a curvature (See **Chapter 4**).

As noted in the next section, hearing analysts have generally attempted to model this classical OC with the top chamber actually filled with a gas. Such modeling results are inappropriate.
It is safe to say that most of the analytical modeling and mathematical modeling up to the present day has been done by those who have not explored the actual physiology of the situation. They have generally been naive, or at best novices in the field (typically graduate students or post-doctorial students with very limited experience). On occasion, they have been led by more senior docents exhibiting a significant leaning toward analogy and particularly allegory.

Much of the modeling has avoided the use of measured values of the relevant parameters and employed overly simplified statements. As an example, "The length of the cochlea is significantly smaller than the acoustic wave length such that effects of compressibility may only become relevant at the highest physiological frequencies[6]." This statement apparently refers to the wavelength of acoustic energy in compression waves in water, not the slowly moving SAW at a fluid/fluid interface. In fact, the OC is longer than the wavelength of all acoustic waves but the lowest frequency traveling along it. The models are invariably overly simplified in order to fit an unreal situation.

W.2.2 The theoretical framework of Lighthill & of Main

Lighthill[7] and Main[8] both address the subject of *gravity* waves as a specific case of the general situation involving one-dimensional waves at a high density fluid/low density fluid interface, specifically a water/air interface. They then extend their model to include the surface tension of the surface of the water. Unfortunately, they use different coordinate systems. Their mathematical solution can be used to address multiple situations by recognizing which terms are relevant in a given situation.

Both authors introduce a number of simplifications to simplify the mathematics involved.

Main begins his presentation by describing his boundary conditions in text form (pages 233-235). What he does not make clear is that his analysis only applies to water in a straight trough with the wave motion generated by a "very special kind of breeze might blow along the length of the canal, exciting a sinusoidal traveling wave." The message is, the equations only apply to a wave traveling perpendicular to a beach and generated by a breeze also perpendicular to the beach. The equations do not apply to a curled trough. Lighthill is more specific, he begins with the description of energy radiating spherically from a simple (point) source (page 17) in accordance with Maxwell's Equations. The energy is described initially in spherical coordinates. In developing his one-dimensional wave

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[6]Edom, E. Obrist, D. & Kleiser, L. (2011) Simulation of fluid flow and basilar-membrane motion in a two-dimensional box model of the cochlea *In* Shera, C. & Olson, E. *eds*. What fire is in mine ears: Progress in auditory biomechanics. NY: Amer. Inst. Physics pp 608-610

[7]Lighthill, J. (1978) Waves in fluids. London: Cambridge Univ. Press

[8]Main, I, (1993) Vibrations and waves in Physics, 3rd Ed. Cambridge Univ Press; the three editions are quite similar

equations, he first describes a wave at a great distance from its source. This wave can generally be described as a plane wave existing perpendicular to its direction of travel. He then restricts his plane in a manner similar to Main where he defines a "shoreline phenomenon of edge waves," The wave is considered unchanging in the direction parallel to a long "beach." In chapters 3 & 4, water waves are initially described that travel on a water/air interface but radiate out in two dimensions. He becomes more specific and delineates acoustic waveguides as hard-walled ducts of constant cross section. The analyses of both Main and Lighthill (pages 207 and 419) apply only to acoustic energy propagating along the longitudinal axis of straight ducts at an interface between a liquid and a gas.

The analyses of both Lighthill and Main define the frequency of a sound wave and its wavelength at a given point in the local media as independent variables. The local velocity of the sound wave as a function of frequency is calculated as the product of these two variables. This situation accommodates the situation where the phase velocity and the group velocity of a wave are not necessarily the same. The result is dispersion and changes in the envelope of the wave with position along the path of the wave.

Patuzzi[9] adopts the analyses of Main implicitly. After citing Sieberts (1974), who famously said, "The influence of the spiral structure of the coclea will be ignored," Patuzzi asserts the character of the cochlear spiral as "not of any great importance, especially at low frequencies where the cochlear dimensions are much smaller than the wavelength of sound in the cochlear fluids." He proceeds with an analysis of acoustic energy projection within a straight cochlea based on a liquid/gas interface. He then notes the velocity of the acoustic energy within the cochlea is not related to the compression waves in fluids (typically 1500 m/s), and adopts propagation via a surface acoustic wave of lower velocity, and subsequently much shorter wavelengths. He initially adopts the equation of Main and introduces a modified form from Patuzzi's colleague, Yates. Neither Patuzzi or Frosch (who applies the work of Patuzzi) discusses a duct system where the fluid is overlaid by a second fluid of similar density.

> Unfortunately, the equations on pages 210-211 of Patuzzi were mangled in the typesetting process. However, they can be reconstructed by reviewing the work of Main.

The set of conditions addressed in figure 4.7 of Patuzzi is not complete. It does not include the condition of a fluid/fluid interface; this is the condition key to the operation of the cochlea.

It is important to note the operation of the cochlea is not dependent on the gravity vector. Psychophysical experiments demonstrate, the auditory modality operates perfectly normally when inverted. Neither does it involve a liquid/free surface as suggested in Patuzzi and in Frosch. The use of a double density fluid in Patuzzi's conformal transformation of the organ of Corti to compensate for the effect of the gravity vector is not valid. Similarly, the analysis of Frosch does not address the actual situation in the Organ of Corti. Medical experiments demonstrate no gas is found in the cochlear fluids (dissolved or as a gas) except in the case of diseases like Meniere's syndrome. Analyses of the operation of the active acoustic system of the dolphin shows that the presence of gas bubbles or gas-filled chambers are of major significance in acoustics. A gas bubble present on the surface of Hensen's stripe, along with a variety of other density discontinuities, can provide a ready explanation for "Kemp's Echo." This echo is recognized to incorporate a long delay (measured in milliseconds rather than microseconds) in its arrival back at the oval window. This long delay is incompatible with an acoustic wave traveling within a factor of one hundred of 1500 meters/sec, the speed of acoustic energy compression wave in water.

The reader is referred to Figure W-1.1-1 in support of the following discussion. Note the implicit gravity vector in frames **A**, **B** & **C** and the potential difference in density applicable to frames **C** & **D**.

Those investigating fluid dynamics frequently work in the domain of pressure as a function of position. In the one-dimensional wave case, the result is pressure versus the position x.
Using Newton's First Law, F = m•accel, the pressure can be expressed as h•y•z•ρ•g where y•z is the unit footprint, h is the height (or head) of the fluid, rho is the density of the fluid

[9]Patuzzi, R. (1996) Cochlear micromechanics and macromechanics *In* Dallos, P. Popper, A. & Fay, R. *eds*. The Cochlea. NY: Springer Chapter 4

and g is the gravity vector. The unit area of the fluid footprint equals the unit area of the pressure footprint.

Dividing both sides of the above equation by the footprint;

$P = F/(y \bullet z) = h \bullet \rho \bullet g$

$P_e = p_a + gh\rho_1$ at a gaseous surface

$P_e = p_a + gh\rho_1(1 - \rho_2/\rho_1)$ for an immersed surface (of a duct).

Using Lighthill notation, the dispersion relationship for three conditions are given in **Figure W.2.2-1** for the straight duct ***that does not apply to the cochlea***. However, the lower equation does explain how the slow velocity of the SAW within the cochlea is achieved.

$$\omega^2 = (g \cdot k + Tk^3/\rho)\tanh(h \cdot k) \qquad \text{liquid/gas interface}$$

$$\omega^2 = (g \cdot k(1 - \rho_2/\rho_1) + Tk^3/\rho_1)\tanh(h \cdot k) \qquad \text{liquid/liquid interface}$$

$$\omega^2 = (0 + Tk^3/\rho)\tanh(h \cdot k) \qquad \rho_2 = \rho_1 \text{ liquid/liquid interface}$$

Figure W.2.2-1 The dispersion relationships for the one-dimensional wave model for various conditions. The top equation applies to a water wave traveling along the axis of an open trough below a gas of significantly lower density. The two lower equations apply to a wave propagating at the interface of two fluids of similar density. These equations do not provide the guiding element required to achieve acoustic energy propagation along a curved path.

The top equation is the textbook variant for a simple water/air interface. The middle equation describes the situation for any liquid/liquid interface (essentially relating to a duct within a water-column. The value of h is taken as the average depth between the interface and the bottom of the water-column. The third variant applies to a liquid/liquid interface where the two liquids have the same density. The density of the endolymph within the scala media is essentially the same as the density of the liquid crystalline material on the first surface of the tectorial membrane.

These equations take on different characters as a function of the value of the argument of the hyperbolic tangent with the argument (h•k). For h•k<<2, the hyperbolic tangent is equal to its argument, typically a number less than 1. For h•k >>2, tanh = 1.00 and the term can be ignored.

The velocity of the traveling wave is equal to c = w/k in all of the above cases. .
Where the tanh term is replaced by h•k, the dispersion relationship varies with the wave number and the velocity of the propagating wave varies with frequency. It is dispersive.

Where the tanh(h•k) term can be ignored, the dispersion relationship is proportional to the square root of the wave number and the velocity of the propagating wave is independent of frequency. It is nondispersive.

For the cases where both the tanh term and the gravity term can be neglected, the only restoring force is surface tension. The resulting waves are called capillary waves. They are the type of interest in hearing over the majority of the length of the Organ of Corti. The wave velocity as a function of wavelength is similar to a hyperbola as illustrated by the left portion of the curve in **Figure W.2.2-2**.

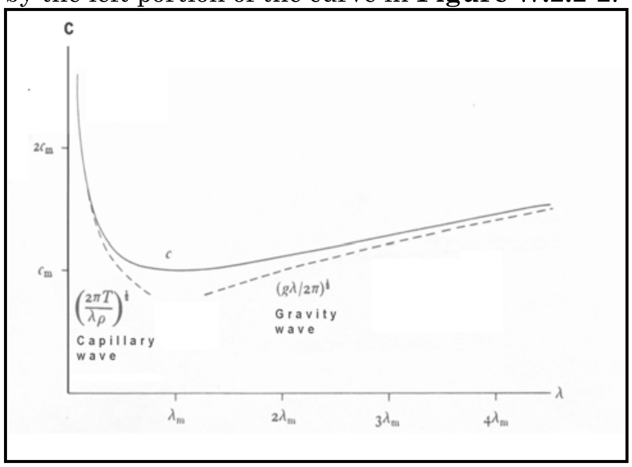

Figure W.2.2-2 The wave speed, *c*, for ripples on deep water. Note the separation into two individual components. The minimum wavelength and speed are designated by c_m & λ_m. From figure 56 in Lighthill, 1978.

Lighthill provided a more detailed **Figure W.2.2-3** for the similar case of shallow water. This appears to be the function used by Ghaffari et al to fit their data points. However, this function applies to ripples (sagittal plane amplitudes) representing Rayleigh waves instead of the assertion by Ghaffari et al. that their waves were lateral plane waves representing Lamb waves.

After exploring the model of Main, Patuzzi explored the condition developed by Yates[10] involving a mass distribution on the surface of a water/air interface and an elastic mechanism introduced between the distributed mass and the bottom of the water. This configuration introduced a resonance that appears to support the conceptual acoustic energy propagation described as, and sometimes labeled, von Bekesy propagation. However, based on the original boundary conditions of Lighthill and of Main, this configuration does not apply to a curved cochlea and may not apply to an immersed interface between two fluids loaded in this way within the cochlea.

Patuzzi did not provide parameters for his elements used in his interpretation of the Yates equation, nor did he provide calibrated vertical axes for his figure 4.7. His explanation of the performance of the model modified to follow Yates is only provided in caricature (figure 4.8).

Frosch[11] cited the work of Patuzzi and of Seibert as what appear to be his major references. His equation (1) is presented *de novo*. It does not appear in the Patuzzi material. He does not detail the circuit that underlies this equation nor the specific stiffness, S, and mass, M, values used in his paper.

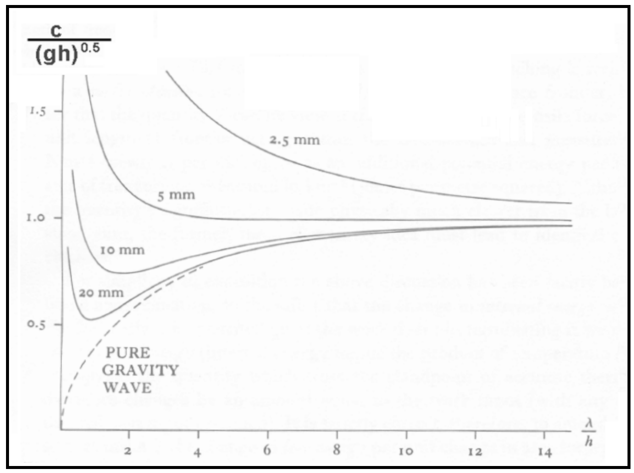

Figure W.2.2-3 Ratio of the wave speed c to the long-wave value for ripples on shallow water of depths *h* equal to 2.5 mm, 5 mm, 10 mm & 20 mm. The pure gravity wave is shown for comparison. From figure 57 of Lighthill, 1978.

W.3 Multi--dimensional waves in fluids

W.3.1 The expansion of de Boer

De Boer[12] followed the conventional approach discussed above in 1996 when he developed a one-dimensional wave model based on a set of assumptions (page262) with the same effect as those used by Lighthill and by Main, he omitted the interaction between the out of plane wave motions (in the y direction) on the in plane motions (projection in the x direction with a signal amplitude in the z direction). He described this as the “classical” model. It is essentially the pre-Maxwellian Equations view of the world. After operating in the one-dimensional model world, de Boer broadens his discussion into a two and then three dimensional world using only semantics. He fails to reintroduce the cross products so critically important to understanding acoustic energy propagation in a Maxwellian propagation world.

[10] Yates, G. (1976) Basilar membrane tuning functions. PhD dissertation, Univ of Western Australia

[11] Frosch, R. (2009) DP phases in mammalian cochleae, predicted from liquid-surface-wave formulas *In* Cooper, N. & Kemp, D. *eds*. Concepts and Challenges in the Biophysics of Hearing. NJ: World Scientific pp 41-47

[12] de Boer, E. (1996) Mechanics of the cochlea: modeling efforts *In* Dallos, P. Popper, A. & Fay, R. *eds*. The Cochlea. NY: Springer Chapter 5

Writing in 2009, de Boer[13] addresses the major differences between the "classical model" that he has written often about, and the real world situation within the cochlea. He notes, "There are at least three categories:"

1. Hidden waves associated with the putative amplification by the OHC.

2. Hidden waves designed as distortion products (DP) when two waves travel simultaneously within a nonlinear medium

3. Hidden waves that he associates with the compressibility of the material within the Organ of Corti.

While these allegorical forms of hidden waves may make his analysis more compatible with the real world, they are not supported by this work, either individually or as a group. An analysis derived from first principles, and not one-dimensional water wave theory, is needed to explain operation of the Organ of Corti.

de Boer does note that the distortion products propagating beyond their point of generation within the Organ of Corti are limited to the "lower sideband DP's"; such an interpretation is in conformity with the limits placed on the OC by the Marcatilli Effect (**Section 9.11.3**).

de Boer then introduces a section labeled, "Classical and non-classical models." He asserts, "In view of the results reported above it seems that we have to consider non-classical models to describe and explain the data. It is difficult to generalize, hence we will mention a few types of such models." After introducing a few papers by long time associates, and both a feed-forward and feed-backward models, de Boer concludes the section saying, "A combination of these two models would be advantageous because each of those, when considered by itself, tends to produce deviant phase shifts that are difficult to reconcile with data." All of his non-classical models are variations on the conventional theme, that the low Q movements of the basilar membrane are the driving force behind the high Q signals generated within the neural system by the Organ of Corti.

Again, an analysis derived from first principles is needed to explain operation of the Organ of Corti. A patchwork, floating model as suggested by de Boer is not acceptable.

W.3.2 The requirement for a wave "guide"

For an acoustical energy wave to follow a curved path, it must be guided. This can be performed by multiple reflections at the boundaries of a container, or it can be obtained by introducing a gradient into the density profile of the medium. In both cases, all of the conditions of Maxwell's Equations must be satisfied.

The analysis in **Section 4.4** of the main text relies upon a variation in the gradient to cause a continual curving of the energy profile associated with the decreasing helical spiral of Hensen's stripe found on the functional surface of the tectorial membrane.

As discussed in **Chapter 4**, the acoustic energy introduced onto the surface of the tectorial membrane follows the defined spiral until it encounters a curvature that is not consistent with propagation of the highest energy component of the wave. That component is stripped off of Hensen's stripe in accordance with Marcatilli's Effect and propagates across the surface of the tectorial membrane until encountering the outer hair cells where it is efficiently absorbed.

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W.3.3 Description of acoustic energy projection within the Organ of Corti

[13] de Boer, E. (2009) Obvious and "hidden" waves in the cochlea *In* Cooper, N. & Kemp, D. *eds*. Concepts and Challenges in the Biophysics of Hearing. NJ: World Scientific pp 34-40

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The key to understanding the operation of the Organ of Corti is to recognize;

1. The acoustic energy of the vestibule is delivered to the tectorial membrane and not the basilar membrane

2. The acoustic energy propagates along the majority of the Organ of Corti in accordance with the density variation of the liquid crystalline Hensen's stripe.

3. The OHC are like IHC; they are strictly sensory neurons.

Both the OHC and the IHC incorporate piezoelectric materials that are bidirectional. They can convert mechanical force (acoustic energy) into electrical signals or electrical stimulation into mechanical motion.

Most of the recent experimental investigations have employed mice. The Organ of Corti in the mouse is about 8 mm long compared to 35-38 mm in human. The very small scale of the Organ of Corti as well as the complete labyrinth, makes it difficult to examine their tectorial membrane in detail over its full length. To provide a more complete description of the Tectorial membrane over its full length, **Figure W.3.3-1** expanded from Anson, will be used. This figure shows the complete scala media of the human with cross-sections attempting to describe the liquid-crystalline surface of the TM at critical locations.

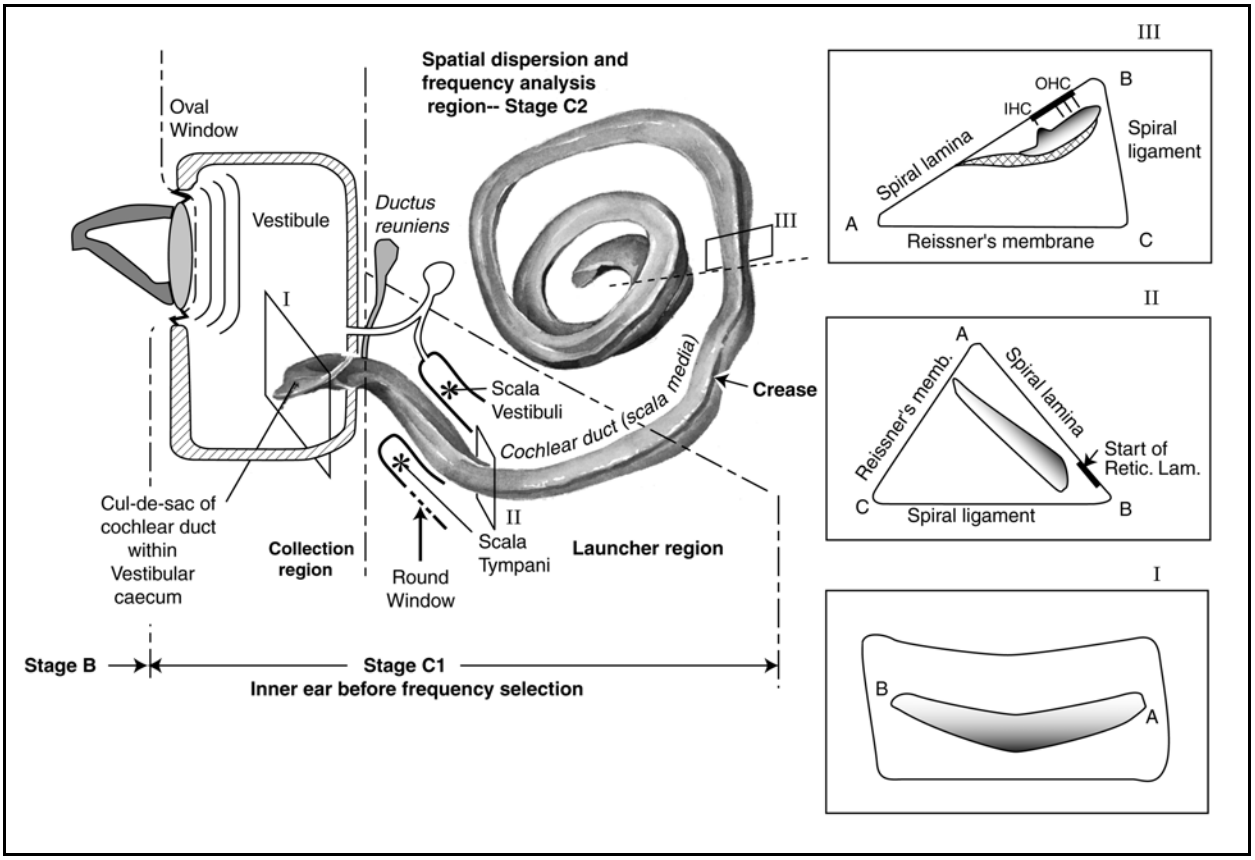

Figure W.3.3-1 Proposed detail configuration of the vestibule and cochlear duct. Inset I; gel surface of tectorial membrane is the only identifiable feature within the duct Acoustic energy enters the duct from the top. The gel acts as a large area (~1 mm^2) second surface SAW launcher. Energy begins to accumulate at the apex of the convex (parabolic) surface. Inset II; the initial section of the cochlear duct twists to maintain a minimum in-plane curvature of the duct while condensing the energy into an area near the future Organ of Corti. The walls of the duct are shown with their eventual labels. They may not be appropriate at this location. Inset III, the gel surface achieves its final shape and location. It is now associated with the remainder of the tectorial membrane acting as an inertial mass. The energy has been concentrated in Hensen's stripe above the IHC. The limited region between the spiral lamina and the spiral ligament forming the basilar membrane is shown by the heavy line. Main figure expanded from Anson, 1992.

Inset I shows the proposed cross-section of the liquid crystalline portion of the TM that extends into the vestibular caecum. This structure will be described further below in its role as the surface acoustic wave launcher used to create a slow surface acoustic wave of the Rayleigh type. The shading represents the acoustic energy shown accumulating on the opposite side of the liquid-crystalline layer from the oval window.

Insert II shows the cross-section of the liquid-crystalline portion of the TM as it travels toward the Organ of Corti but before, the cochlear partition begins its in-plane spiral that causes the dispersal of the acoustic energy as a function of frequency. The acoustic energy is shown accumulating on the surface of the liquid-crystalline layer in the vicinity of where the sensory neurons will be encountered farther along the scala media.

Insert III shows the cross-section of the liquid-crystalline portion of the TM within the active region of the Organ of Corti. The acoustic energy has been accumulated within the topographic waveguide known as Hensen's stripe prior to its frequency selective (place-dependent) dispersion.

W.3.4 Protocols for exploring surface acoustic waves at the gel-fluid interface of the TM

Sections M.2.4 and **M.2.5** have provided a framework for laboratory investigations of acoustic energy propagation on the gel-fluid interface of the TM. It should be clear that the gel-fluid interface can support a number of acoustic energy propagation modes and that uncontrolled boundaries can cause multiple reflections of these modes. The result can be a confusing set of intersecting waves propagating uncontrollably across the surface of the interface. Morse & Ingard illustrate a variety of these possibilities for a mechanical membrane (pages 206-216 of 1986 edition).

Ohm & Hamilton have discussed the propagation of Rayleigh waves in thin structures similar to the TM. They note (their figure 2 and accompanying text) that for thin coatings, where the wavelength of the propagated energy is much longer than the thickness of the coating, most of the higher order modes are suppressed and only the first Rayleigh mode need be considered.

Figure W.3.4-1 presents a theoretical model of the tectorial membrane beginning with the simplest presentation. It is based on figure 1 of Ohm & Hamilton. The left frame shows a totally theoretical uniform density TM with a uniform density gel coating between the bulk TM and the uniform endolymphatic fluid. All dimensions of the model extend to infinity to eliminate any reflections of the energy introduced into the model at boundaries. The right frame shows two practical modifications of the theoretical model encountered in the real TM. First the semi-infinite substrate is truncated by an absorber of matching impedance that will not support the reflection of energy. This absorber is analogous to the covering net of the real TM. As long as the impedances are matched, it is not necessary for the covering net to be parallel to the gel coating. Second, the gel coating is truncated on the right by an absorber of matching impedance that will not support the reflection of energy. This absorber is analogous to the marginal net of the real TM.

The coordinate system shown is arbitrary. There is no requirement that the energy, indicated by the vector k, flow parallel to the x-axis. In the real TM, there are lenticular fibrils (micelles) arranged near the surface of the substrate material that are believed to be parallel to the direction of energy flow. In the real TM, the x-axis is generally labeled the radial axis when examining small samples of the TM. Under those circumstances, the x-axis should be taken as perpendicular to the axis of the marginal net and the y-axis can be considered the longitudinal axis that is parallel to the marginal net.

In the theory presented in this work, the energy propagates across Kimura's membrane only in the positive x-direction due to the means by which the acoustic energy is introduced into the surface between the gel and the fluid (Marcatili's mechanism).

It is important to note the following. If the model shown on the right is correct, the TM of hearing operates as an analog of a semi-infinite bulk material and not as a thin plate. It is important to develop any detailed mathematical model of the TM in accordance with whether it is a semi-infinite volume or a thin plate. The mathematics are quite different for the two cases.

The predicted space constants are so large for the surface acoustic waves traveling on the gel-fluid surface of actual TM materials, it is difficult to measure them in small samples of actual TM material. It may be necessary to attempt the measurement on full longitudinal samples of the interface.

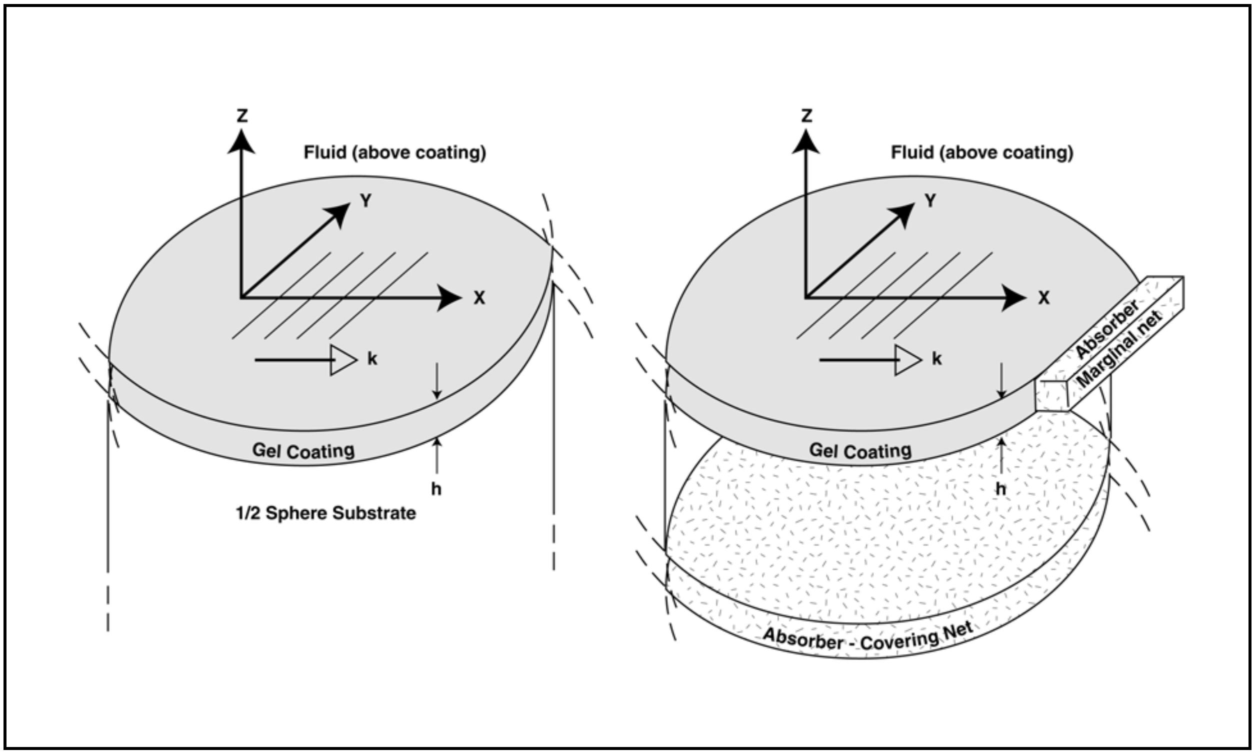

Figure W.3.4-1 A theoretical acousto-mechanical model of the tectorial membrane. Left; the fundamental model of a gel coating on a semi-infinite substrate supporting the propagation of a surface acoustic wave at the gel-fluid interface. The density of the gel coating is ρ_1, that of the endolymph, ρ_2. Right; a description of the real tectorial membrane with a real absorber of matching impedance, the covering net, replacing the semi-infinite substrate. An additional absorber of matching impedance, the marginal net, is shown replacing the infinite extent of the gel coating on the right side.

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